On the Construction and Analysis of Multiple Input Multiple Output Subnet Structures

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Abstract

In this paper, the construction and analysis of multiple input multiple output (MIMO) subnets is presented. Hierarchical Time-Extended Petri Nets (H-EPNs), allow for the generation of MIMO subnets through bottom-up synthesis techniques similar to those discussed in PN literature [3]. A simple transformation of these MIMO subnet structures into equivalent single input single output (SISO) subnets is discussed. This approach allows for the development of a hybrid approach to systems modeling using H-EPNs where the top-down decomposition methods may be easily integrated with the bottom-up synthesis methods.

1. Introduction

Petri nets (PNs) are a graphical and mathematical modeling tool. The PN representation of a system consists of places and transitions (represented as a circle and rectangle, respectively, in a PN representation), with tokens flowing along the arcs interconnecting them. These tokens are used to simulate the dynamic and concurrent activities within the system. As a mathematical tool, PNs are used to describe the behavior of the system they represent, as state equations and algebraic equations. PNs and their modifications have been proven to be useful for the modeling and analysis of several classes of systems, including computer systems, software, communication networks, production/process control systems, knowledge-based systems, and manufacturing systems. Murata, in his tutorial-review paper on PNs [2], provides a thorough review of the PN history and their application areas considered in the literature.

There exists one basic disadvantage related to the applicability of PNs for the modeling and analysis of complex systems: the problem of state-space explosion (also referred to as PN complexity). Even for a moderately sized system, the complexity of the design makes the PN model unreasonably complex to handle. Thus, the problem of systematically constructing a PN model that has desirable properties, surfaces.

Generally, PN modeling techniques deal with the problem of PN state space explosion either by top-down decomposition methods, that is, by means of subnet abstractions, or by bottom-up techniques that preserve overall system properties once the properties of individual subnets have been established. A top-down decomposition approach leads to the definition of subnets and subnets are defined and developed such that they do not violate the properties of the overall net. A bottom-up approach to the development of a PN system model will lead to the creation of PN structures that are not essentially SISO. Research on PN extensions to systems modeling / analysis have focused on the use of SISO subnet structures because they are easy to develop and analyze. More on these approaches may be found in [1, 5] and references therein. Though these two approaches have been discussed independently in great extent in PN literature, there does not exist a PN extension that will accommodate model development and analysis of MIMO nets. The H-EPN approach to PN modeling addresses this issue by adding the notion of activator arcs and modeling independent subnet initiations (separately defined subnets as well as independent subnets within a MIMO subnet) by means of conjugate places. The idea of using activator arcs has been derived from using Extended Petri Nets (EPNs) in the modeling and analysis of materials handling systems [4]. The advantages of activator arcs have been examined closely with respect to modeling issues [3]. Activator arcs are advantageous when multiple subnet/operation initiations are required. One such application may involve the need to run different algorithms / evaluation strategies on same sets of data.

H-EPN MIMO subnets are defined by means of a bottom-up approach to PN model generation, similar to various other work in literature. The SISO subnets that are independently defined are combined through common places, transitions and arcs to define a MIMO subnet. Such a MIMO subnet is then transformed to a corresponding SISO subnet for easy integration (as a SISO subnet) into a top-down decomposition of the overall system model. The advantage of this approach is that already established analysis techniques can be easily adapted to be used within this framework. Moreover, it also simplifies the generation of highly complex PN structures. The extensions/modifications simplify the modeling and analysis of any hierarchically decomposable system. The generalizations/modifications are related to the following:

- i) Five different types of places are defined,
- ii) Two different types of transitions are defined,
- iii) Two different zero-weighted arcs are defined. These are the activator and inhibitor arcs. More on the arc extensions may be found in [3, 4], and,
- iv) Two different types of tokens make up the graphical system model, referred to as the solid and dotted tokens, respectively.

The graphical representation of the places and arcs in a H-EPN design is shown in Figure 1a. The *ss* place (Figure 1b) is essentially a specialized subnet place of two immediate transitions and four status places. The dotted arcs represent the connections that are used to study system properties during simulation. This structure for the *ss* place implicitly adheres to the property that every source place has a corresponding sink place in the net, thereby allowing for token conservation (that is, all tokens that enter the system, exit the system). This factor is important for ensuring the property of system boundedness.

As an individual entity, a subnet place is not live; its liveness is dictated by the dynamic flow of tokens in the net. Such a property is called "*quasi-liveness*". A *ss* place is required to test the properties of a quasi-live subnet. Decision, action or status places are always the entry places to, and exit places from a subnet. A *ss* place is useful for analyzing the properties of subnet places, since they can be used to study individual subnet properties as shown in Figure 1c. As mentioned earlier, the restriction that the subnet place be a SISO place is relaxed, thereby allowing for multiple points of entry to and/or exit from the subnet. In such a case, every point of entry is associated with a corresponding point of exit. That is, individual SISO nets which share common places, transitions or arcs are combined to form H-EPN MIMO nets and this is similar to the bottom-up synthesis of PN models.

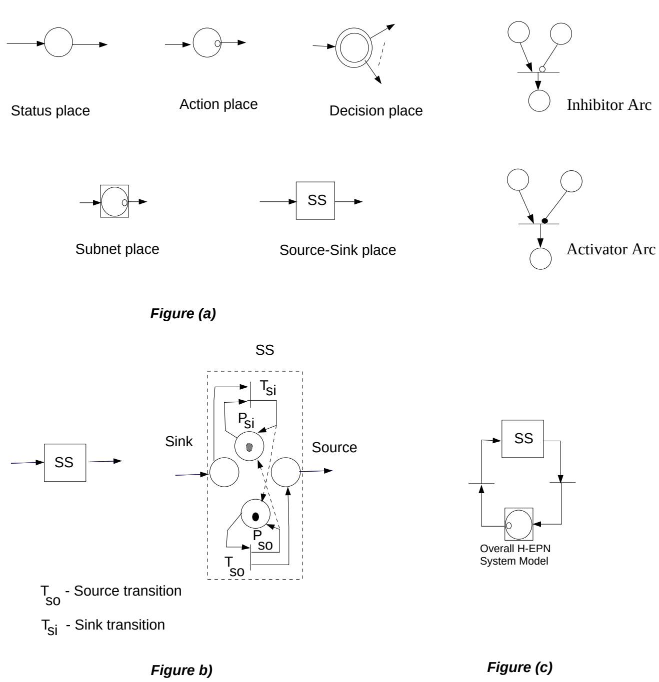

Figure 1. ***Places in an H-EPN***

MIMO-SISO transformation and properties are discussed in section two. Section three concludes the paper.

2. MIMO-SISO Subnet Transformation and Properties

In H-EPNs, MIMO subnets are created due to operation abstraction with respect to the resources used. This definition of a subnet place also implies that similar operations performed at different system areas may be grouped together as a single subnet place. The status of the initiation of such an operation (represented by a subnet place) is maintained by a conjugate place. Thus, although a subnet place may lead to the firing of different transitions after the completion of associated operations, the use of the conjugate place aids in resolving such conflicts. The conjugate place provides for context sensitivity in subnet initiations. This means that different instantiations of the same MIMO subnet corresponding to the respective SISO subnets that form the MIMO subnet are possible and information regarding a particular instantiation is maintained by means of the conjugate place at the higher level net. Figure 2b illustrates a simple MIMO subnet generated by a shared resource place, p_r and Figure 2c shows a more complex MIMO subnet generated by shared places, p_r and p_q, where p_r is a shared resource and p_q sequences the operations of the two SISO subnets of the MIMO subnet. It is to be noted that when such a MIMO subnet place is used at a higher level of abstraction, the two individual SISO nets do not essentially get activated right after one another. They occur in sequence but other operations represented at the higher level net or by other MIMO nets may be inter-spaced between these two operations. Moreover, when they are not initiated alternatively by a higher level net deadlock will occur. Although providing greater modeling flexibility, this approach to PN modeling necessitates greater care while designing higher level nets and corresponding lower level MIMO subnets. It is to be noted that when we talk about subnet liveness we actually refer to the quasi-liveness of subnets.

Before proceeding let us define some of the basic terms used in the rest of this paper.

Definition 1: *Well Formed Subnet (WFS):* A well formed subnet is a SISO marked graph that is bounded, live and reversible. The net in Figure 2a is a WFS.

Definition 2: *Successor ($\sigma(t_i)$):* A transition t_j is said to be a successor of a transition t_i, $\sigma(t_i)$, if there exists a place, p_k, or a set of places, p_k .... p_{k+r}, such that:

$$(t_i)\bullet \;=\; p_k \;=\; \bullet(t_{i+1})$$

……

$$(t_{i+r})\bullet \;=\; p_{k+r} \;=\; \bullet(t_j)$$

$$Thus,\ t_{i+1} \;= \sigma(t_i)$$

$$and \quad t_{i+r} \;=\; \sigma^*(t_i), \qquad \textbf{(1).}$$

$where\ the * indicates\ that\ the$ $\mathbf{distance}$ $is\ greater\ than\ a\ unit,$ $that\ is,\ the\ transitions\ are\ not\ immediate\ succesors\ but\ are\ probable\ distant\ successors.$

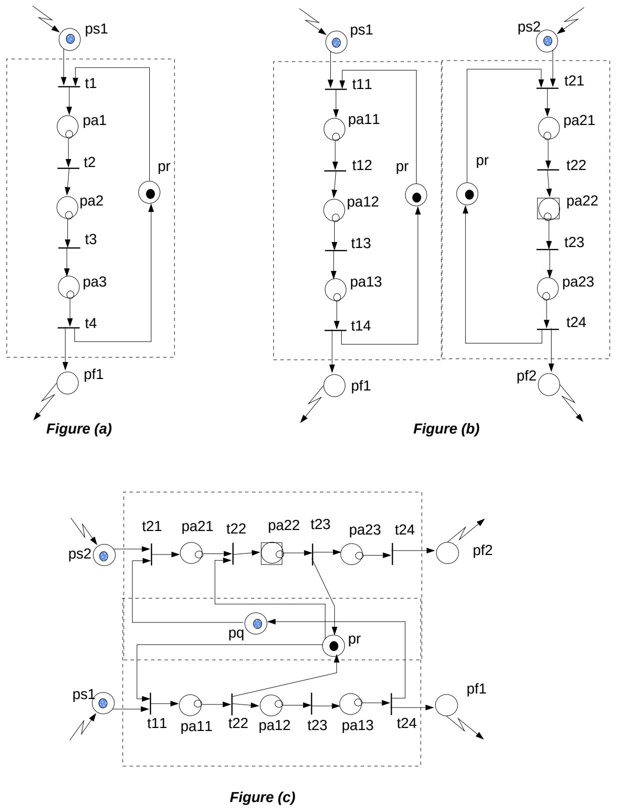

Figure 2. ***SISO/MIMO PN Examples***

Note: Observe that if a net is reversible, then $\sigma^*(t_i) = t_j$ and $\sigma^*(t_j) = t_i$.

Definition 3: *Well Defined Block (WDB):* A WDB is a SISO subnet place, p_{su}^{j} such that if a *ss* place p_{ss}^{i} is introduced such that equation (2) is true, then the combined net is live, bounded and reversible. Thus, *a well formed subnet is a WDB.* Figure 2a is a example of a WDB.

$$\begin{aligned}&Let\ p_s\ and\ p_f\ be\ the\ first\\ &and\ last\ places\ of\ the\ subnet\ place\ \ p_{su}^{j}\\ &\bullet(p_s)\ =\ t_f\ =\ (p_{ss}^{i})\bullet \qquad and\\ &(p_f)\bullet\ =\ t_s\ =\bullet(p_{ss}^{i})\end{aligned} \qquad \textbf{(2).}$$

Definition 4: *Interacting Subnet (IS):* An interacting subnet is made of two or more WDB's p_{su}^{j} $1 \le j \le n;\ n \ge 2;$ n being the number of WDB's sharing the same set of resources.

Definition 5: *MIMO Subnet:* If the individual WDBs of an interacting subnet of two or more WDB's can be ordered such that: *(i)* equation (3) is satisfied, and, *(ii)* the combined net is live, bounded and reversible, then it is called a well formed MIMO subnet. In such an ordering, a set of *ss* places p_{ss}^{i} $1 \le i \le n;\ n > 2;$ are introduced to form a combined net with the interacting subnet. Figures 2b and 2c are examples of well formed MIMO subnets.

$$\begin{aligned}&Let\ p_{sj}\ and\ p_{fj}\ be\ the\ first\\ &and\ last\ places\ of\ the\ subnet\ place\ \ p_{su}^{j}\\ &\bullet(p_{s1})\ =\ t_{f1}\ =\ (p_{ss}^{1})\bullet \qquad and\\ &(p_{f1})\bullet\ =\ t_{s2}\ =\bullet(p_{ss}^{2}),\\ &.....\\ &\bullet(p_{sj})\ =\ t_{fj}\ =\ (p_{ss}^{j})\bullet \qquad and\\ &(p_{fj})\bullet\ =\ t_{si}\ =\bullet(p_{ss}^{i}),\\ &.....\\ &\bullet(p_{sn})\ =\ t_{fn}\ =\ (p_{ss}^{n})\bullet \qquad and\\ &(p_{fn})\bullet\ =\ t_{s1}\ =\bullet(p_{ss}^{1})\end{aligned} \qquad \textbf{(3).}$$

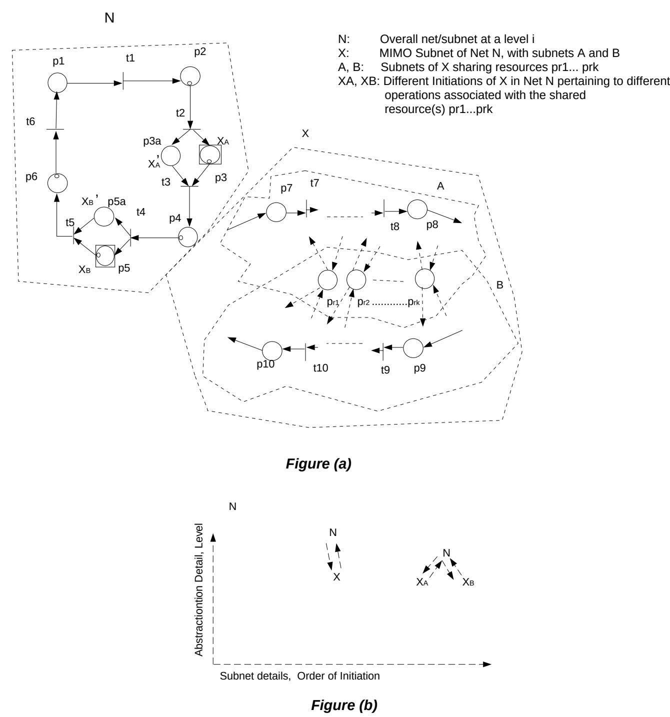

Figure 3. ***MIMO Net structure***

Definition 6: *SU-Connection (SUC):* A SU-Connection (refer Figure 3a) (subnet connection) is said to be established between a net N and a MIMO subnet $X_{mimo} \in N$, when a WDB, $p_{su}^{j} \in X$ and its conjugate place in N are marked by a transition firing in N.

Definition 7: *Subnet Activation Time* (${}^{N}t_i(X_A)$)*:* ${}^{N}t_i(X_A)$, represents the i th instance of activation of a subnet X_A in net N.

Definition 8: *QR Set:* A transition, t_i, is said to be associated with a QR set, if there exist a set of disjoint places P_Q and P_R such that P_Q is the set of places which inhibit t_i and P_R is the set of places which activate t_i.

Definition 9: *Dependent and Independent WDBs:* Suppose there exist 3 WDB's X_A, X_B and X_C belonging to a MIMO subnet X_{mimo} of a net N.
Then, X_C is an independent WDB and X_A, X_B are dependent WDBs if:

$$\forall\, p_i \in X_A,\; p_j \in X_B,\; p_k \in X_C$$
$$X_C \cap X_A = X_C \cap X_B$$
$$= P_{ri},\; 1 \le i \le s,\; s \le k,\; and$$
$$X_A \cap X_B = P_{rj},\; 1 \le j \le k,\; and$$
$$\exists (p_m) \in P_{rj}:$$
$$1.\quad \bullet(p_m) \in X_A, \qquad \textbf{(4).}$$
$$2.\quad (p_m)\bullet \in X_B,$$
$$3.\quad (p_m) \notin P_{ri},$$
$$Where,$$
$$s: \#\, resources\ shared\ by\ the\ WDBs,$$
$$k: Total\ \#\, places\ shared\ by\ the\ WDBs.$$

That is, X_C is not affected by any shared places if all the shared resources are available for its initiation, whereas X_A and X_B are sequenced by a shared place.

The following are then observed from the above definitions regarding the resource, $p_r \in P_{ri}$, $1 \le i \le s$, and p_m:

- p_r is used in the operations of Net N outside of X if X_A, X_B are temporally spaced in N.
- p_m is used in the communication between X_A, X_B when they are triggered alternatively or simultaneously (that is, $\bullet(X_A) = \bullet(X_B)$) in N.

Moreover, it can be noticed that the MIMO subnet in Figure 2b consists of 2 independent subnets while the MIMO subnet of Figure 2c consists of two dependent subnets, which share a common place p_q that determines the liveness of the MIMO subnet, and hence the overall net.

Consider the net in Figure 3. Let X be the MIMO subnet of a net N consisting of 2 WDBs X_A and X_B such that X_A and X_B are initiated by N at different time instances. Let X_A and X_B encapsulate a resource represented by a status place $p_{ri} \in X$.

The following facts can then be observed / established:

Fact 1: A MIMO subnet is made up of two or more WDB's.

Fact 2: If a resource (or a set of resources) represented by a place p_{ri} is required in a higher level net (Net N in Figure 3) then there exists at least 2 WDB's X_A and $X_B \in X$, $X \in N$ and $X \in P_{su}$ which oversee the request and release of the resource(s) when resource failure identification is built into the request and release operation.

Theorem 1: *A ss place is a WDB.*
Proof: The proof of this theorem is trivial. This is because if two *ss* places p_{ss}^i and p_{ss}^j are combined using two temporary transitions t_s and t_f then one of the *ss* places, say p_{ss}^i, can be considered to function as a WDB connected with a *ss* place, p_{ss}^j. Equation 2 holds for this combined net.

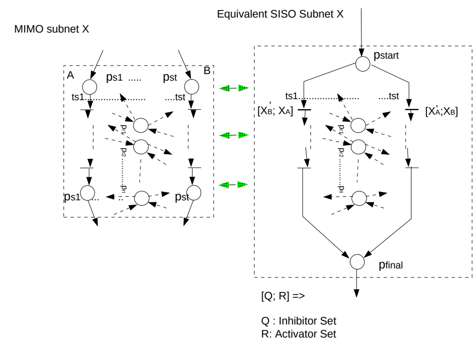

Figure 4. ***MIMO — SISO Translation***

Theorem 2: *For every well formed MIMO subnet, say X_{mimo}, there exists a well formed SISO subnet, X_{siso}, that represents the same set of operations, such that operations corresponding to the individual WDB's in X_{mimo}, say h, correspond to h branches that are activated due to the firing of one of the h transitions that are output to the single input place, p_s, to X_{siso}.*
Proof: It is to be recalled that H-EPNs allow the use of inhibitor and activator arcs. The MIMO-SISO transformation is a 2–step process as illustrated in Figure 4:

- ***Creating a SISO structure***: Remove all the input places of the WDBs and replace them by a single decision place, p_s, with output arcs running from the decision place to all the output transitions, say $t_h \in T_h$, (T_h is a set of transitions) of the places removed . Remove all output places from the individual WDBs and replace them by a single place p_f which will be the final output place for all the WDBs. The transition that this place will fire at the higher level net on completion of the subnet operation will depend on the conjugate place that was simultaneously marked
- ***QR set development***: For each transition in T_h, that is output to the place p_s, the set Q consists of all the places that inhibit the transition from firing, and the set R consists of the corresponding conjugate place (places with implicit activator arcs to the corresponding transtions) at the higher level net. Sets Q and R may also contain other constraints that can affect the selection of the particular WDB of the MIMO subnet.

Thus, these new transitions with QR sets will have two more conditions to satisfy before they are enabled [3]. These conditions correspond to the activating and inhibiting places (all such places may not necessarily be conjugate places at the higher level net, they may also be other places at the higher level net) that are associated with the corresponding transitions.

While using PN simulation tools that do not allow the use of activator arcs, for simulation purposes, the following approximation technique can be used. The token from the corresponding conjugate place will be removed (by means of an ordinary arc definition from the conjugate place to the corresponding transition) thus activating the WDB and then reintroduced when the last place of the WDB is marked. Thus, this will involve a simple graphical modification of the H-EPN system model.

Theorem 3: *Let N be the net that contains the MIMO subnet X_{mimo} with a correct initial marking μ_{N_0}. Let X_A and X_B be WDBs of X that are SUC in N. Let S be the net obtained after the SUC of X_A (or X_B). Then:*

- *S is bounded ⇔ N is bounded.*
- *S is live ⇔ N is live.*
- *S is reversible ⇔ N is reversible.*

Proof: Let μ_{S_0} be the correct initial marking of S such that for every place $p_i \in P_S$, $\mu_{S_0}(p_i) = \mu_0(p_i)$ and for every place $p_i \in P_N$, $\mu_{N_0}(p_i) = \mu_0(p_i)$.

Let X_A and X_B subnets in $X_{mimo} \in N$, and let X_A' and X_B' be the corresponding conjugate places in N. Let p_{sa} and p_{sb} be the starting places of X_A and X_B in X_{mimo}. Let $[Q_a, R_a]$ and $[Q_b, R_b]$ be the QR sets of $(p_{sa})\bullet$ and $(p_{sb})\bullet$ in a corresponding SISO transformation.

By Definition 5, X_{mimo} is bounded, live and reversible. Two cases may be distinguished. The WDBs $X_A, X_B \in X_{mimo}$ may be either dependent or independent WDBs.

Case 1: $X_A, X_B \in X_{mimo}$ are independent WDBs. In this case, there do not exist places in X_{mimo}, or the equivalent X_{siso}, that sequence the subnets X_A and X_B.

$$\begin{aligned}
&Let,\ P_A = P_{X_A},\ P_B = P_{X_B} \\
&and\ P_X = P_A \cup P_B, \\
&N = P_N \cup T_N,\ and \\
&S = P_N \cup T_N \cup P_X \\
&\forall\, p_i \in P_S \cap P_A,\ \mu_{S0}(p_i) = \mu_{A0}(p_i) = \mu_0(p_i) \\
&\forall\, p_j \in P_S \cap P_N,\ \mu_{S0}(p_j) = \mu_{N0}(p_j) = \mu_0(p_j) \\
&\mu_0(p_i)\ and\ \mu_0(p_j)\ are\ the\ correct\ initial \\
&markings\ of\ X_A\ and\ N\ respectively.
\end{aligned} \qquad \textbf{\textit{(5).}}$$

Thus, since, X_A, X_B are independent WDBs

a. S is bounded ⇔ N is bounded.
b. S is live ⇔ N is live.
c. S is reversible ⇔ N is reversible.

Case 2: $X_A, X_B \in X_{mimo}$ are dependent WDBs.

a. N is live, bounded and reversible implies that:

- X_A, X_B have an ordering (A, B), that is, ∃ a sequence of transitions and places, t_i, $p_{su}{}^a$, t_{i+1}, p_{i+1}, ...t_j, $p_{su}{}^b$ in N, such that:

$$\begin{aligned}
&\bullet(p^a_{su}) = \bullet(X'_A) = t_i \\
&(p^a_{su})\bullet = (X'_A)\bullet = t_{i+1} = \bullet(p_{i+1}) \\
&\ldots\ldots \\
&\quad t_j = \bullet(p^b_{su}) = \bullet(X'_B).
\end{aligned} \qquad \textbf{\textit{(6).}}$$

- Thus, ${}^N t_i(X_A) < {}^N t_i(X_B)$ and ${}^N t_{i+1}(X_A) > {}^N t_i(X_B)$.
- $X_A' \in Q_b$, $X_B' \in Q_a$, $X_A' \in R_a$, and $X_B' \in R_b$ in the equivalent X_{siso} transformation.

Thus, the SUC in N alternates between SUC_A and SUC_B. Thus, S is bounded, live and reversible.

b. To prove that N is bounded, live and reversible if S is bounded, live and reversible refer to Equation 5. S consists of places and transitions from N and X_{mimo}. If S is bounded, live and reversible then X_{mimo} is bounded, live and reversible. And $\mu_{S0}(p_i) = \mu_{N0}(p_j) \cup \mu_{A0}(p_k)$, where, $p_i \in P_S$, and $p_j \in P_N$ and $p_k \in P_A$. Thus, if $\mu_{S0}(p_i)$ gives a bounded, live and reversible marking for net S, then $\mu_{N0}(p_j)$ gives a bounded, live and reversible marking for net N.

3. Conclusions

A MIMO-SISO transformation has been established in this paper which provides flexibility to systems modeling with H-EPNs. The analysis of MIMO nets are simplified by the MIMO-SISO transformation that is easily provided by the H-EPN structure.

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